CUBISM AND *CABRI*

Adrian Oldknow shows what is possible with *Cabri* by using it to model some of his holiday snaps from Prague.

Prague, as many people will know, is a beautiful and very accessible city with many architectural gems. One of its lesser-known features is its architectural link with cubism.[1] These include a remarkable streetlamp hidden in a small square, Jungmannovo namesti, just a stone's throw from the main Mustek metro station at the top of Wenceslas Square (Vaclavske namesti). This article illustrates some of the ways in which we can use 2D and 3D dynamic geometry software, in this case *Cabri*, to model the structure on the basis of some tourist photographs.

The base is a seat with a plinth bearing a stack of 5 identical solid geometric objects. How would you describe these objects? Their top and bottom (hidden) faces are hexagons and the 12 sloping faces are trapezia. The central section parallel to the top and bottom faces is a (larger) hexagon. Maybe it would be easier to regard each of these objects as being made from two smaller pieces, each of which has a hexagonal base and six sloping trapezia faces, with a (smaller) hexagonal face parallel to the base? Clearly words are not the best

way of describing such an object – so let's get into some 3D modelling using *Cabri*.
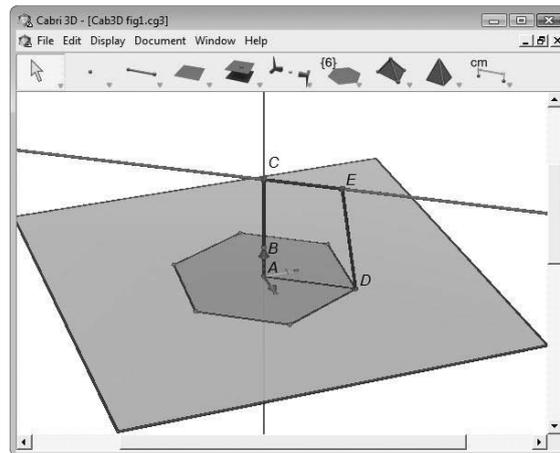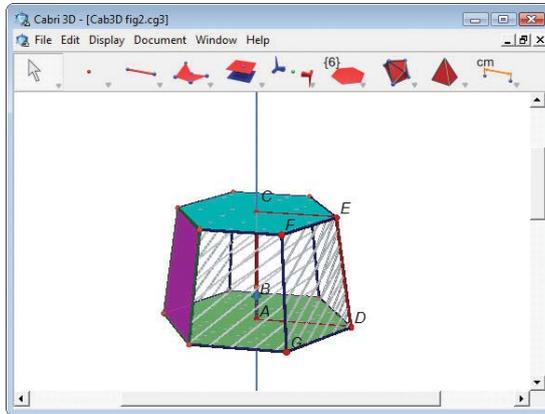

Figure 1

In *figure 1* we are using the 'ground plane' which *Cabri 3D* provides. The origin has been labelled *A* and the point above it labelled *B*. The line *AB* has been drawn and a point *C* constructed on it. The point *D* is any point in the ground plane, and the segment *AD* has been drawn. The line through *C* parallel to *AD* has been drawn and *E* is a point on it. The segments *AC*, *CE* and *ED* have also been drawn. The regular polygon tool has been used to construct a hexagon in the ground plane with centre *A* and *D* as a vertex. So now we have the framework established – and the points *C, D, E* can be slid to change the dimensions of the basic object. In order to define it as a (convex) solid (polyhedron) we just need to construct the top face as the hexagon in the plane through *C* parallel to the ground plane with *C* as centre and *E* as a vertex. Then we can use the polyhedron tool to construct the solid with the two hexagons as opposite faces.

A right-click on an object gives a pop-up menu from which you can adjust the properties of the object (*figure 2*). In our case, the new polyhedron has been shown with faces coloured with
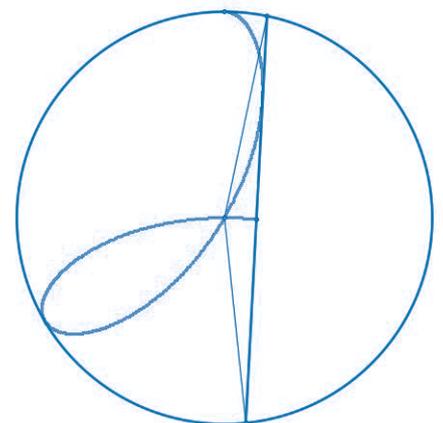
small hatches, so we can see *through* the object. Each of the sloping faces, like *DEFG*, is a trapezium, and one of the faces has been coloured in to make this clearer. Using the right mouse button, you can drag the view and look at the object from different angles.

Figure 2

We can measure lengths of segments, such as *AD*, areas of polygons, such as the hexagonal faces, and volumes of polyhedra, such as our basic lamppost building block (*figure 3*). In order for these to represent the actual Prague streetlamp, we will need to make sure that we match the lengths to measurements from the actual streetlamp.

Figure 3
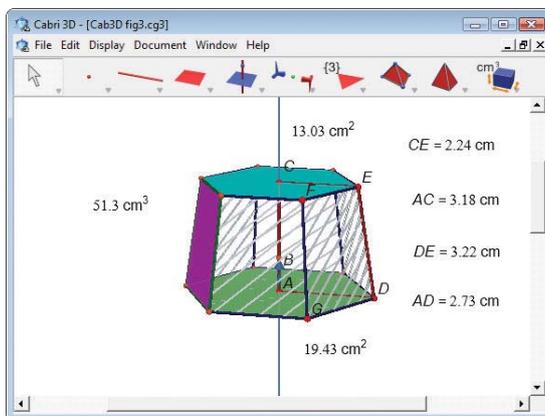
In order to take measurements from the original photograph, we need to import it as the background in a 2D geometry file, such as in *Cabri*

Geometry II Plus. Here the photo is first cropped in a picture editor, and then saved as a bitmap (bmp) file. In *Cabri II*, the picture is imported as a background image – and if it is attached to an object, such as a segment, it can be easily resized. The points *X* and *Y* define a segment, and the point *Z* is on *XY*. The segment *XZ* is constructed, and *XY* is hidden. With a right-click on *XZ*, you can import an image from a file, which can then be resized by sliding *Z*. Knowing (or estimating) the height of the person in the picture, we can work out the scale of the photograph. In this case, it is roughly 30:1. So then we can estimate other measurements, such as the overall height of the streetlamp (about 4.72m), and the combined height of the 10 'building blocks' we are modelling. Of course, there will be some parallax errors, and we must be cautious of spurious accuracy, but we have a rough idea that the vertical height of each block is around 25cm. By zooming in, we can see that the photograph is taken parallel to one pair of sides of the hexagonal faces. So we can estimate their separation using distances such as *GH* and *JK* (*figure 4*). Using some transformation geometry and constructions in *Cabri II*, we can draw scale models of the smaller and larger hexagonal faces and estimate their side lengths (and hence radii *a* and *b*). We can also measure (or calculate) the area enclosed by each hexagon and convert these to the full measurements. In order to estimate the volume of each building block, we can treat it as the difference left when the top part (represented by triangle *KMN*) is removed from a hexagonally-based pyramid (represented by triangle *HLN*). Using the formula for the volume of a pyramid (⅓ × base area × height), we can thus find an approximation for the volume of the building block as about 30 litres. The proper name for such a slice of a pyramid, or cone, is a *frustum*.[2]

Let's see how this works if we return to *Cabri 3D* (*figure 5*). Because the units are quite small, it helps to change the display to show at 'scale 2'. So we can see that with *C*, *D*, *E* set to give the same lengths for *AC*, *AD* and *CE* (as in the *Cabri II* model) we do indeed get values for the length *DE*, the areas of the hexagonal faces and the volume of the frustum which agree well with our estimates. So we also have an alternative way to build the block in *Cabri 3D* by slicing through a pyramid. The point *A* is the origin and *D* is any point in the ground plane (hidden). The green regular hexagon is constructed in the ground plane with *A* as centre and *D* as vertex. The perpendicular to the ground plane through *A* is drawn, and points *C* and *N* constructed on it. The hexagonal pyramid is
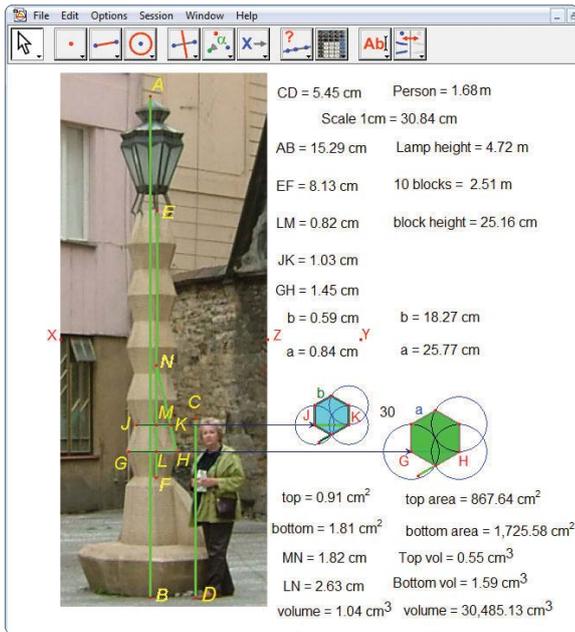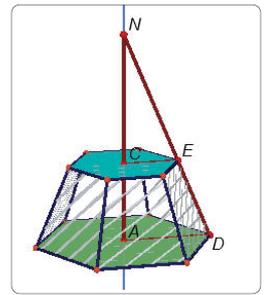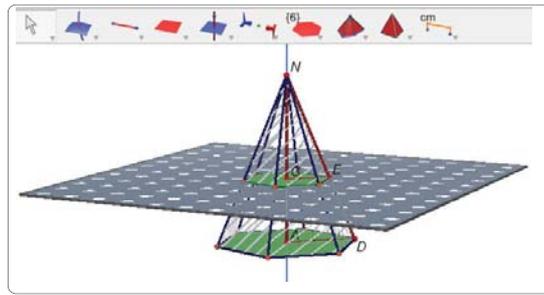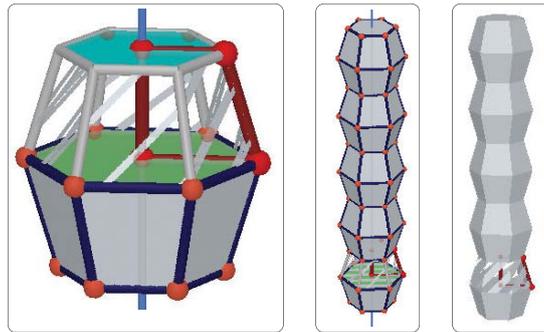
Figure 4

Figure 6

Figure 7

Figure 5

This article has set out to show how the functions of dynamic geometry software – in this case *Cabri 3D* and *Cabri II Plus* – can be used to help model structures both to give good fits to images taken from photographs and to perform analyses on them using measures, scales and calculations. Since the model is defined using 'slideable' parameters, we also have the ability to undertake 'projective modelling; ie, to investigate a range of possible models of objects yet to be built physically.
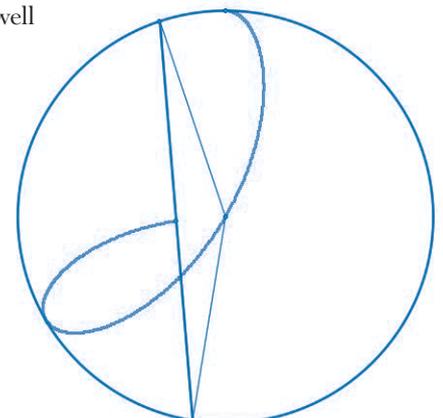
Here are some problems you might like to explore yourself:

a) If a frustum has similar parallel faces of areas A and B a distance h apart, find a formula for the volume of the frustum in terms of A, B and h.

b) If concrete has a density of about 2300 kg/m^3, make an estimate of the mass of the 10 frusta forming the major part of the streetlamp.

c) Make a model of the lamp itself. Using estimates for the density of iron and glass, make an estimate of the mass of the lamp unit.

d) Make a model of the base of the lamp-post. Hence estimate the total mass of the street lamp.

e) Make a booking to visit Prague and be sure to take a tape measure as well as a camera with you!

defined by the hexagon and the point *N*. The plane perpendicular to *AN* through *C* cuts the segment *DN* at *E*. The top hexagon is defined in this plane with centre *C* and vertex *E* (*figure 6*).

Using the 'cut polyhedron' tool from the 'polyhedron' menu on the tool bar, we can slice the pyramid and retain the frustum between the hexagons as the convex polyhedron representing our chosen building block. You can now slide points *C, D, E* to agree with our scale measurements and check that the resulting volume is still in agreement. If we reflect the frustum in its base hexagonal face – the green one – we now have two identical frusta which together make up one of the five bifrusta which form the major part of the lamp-post. By continued reflections we can build this model up in *Cabri 3D* and then adjust the way the polyhedra are displayed to hide the edges and vertices (*figure 7*).

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Notes

1 www.architectureweek. com/2002/0501/ culture_2-3.html

2 This comes from the Latin for 'a piece broken off': see, eg, http://en.wikipedia.org /wiki/Frustum.

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