Mathematics from still and video images
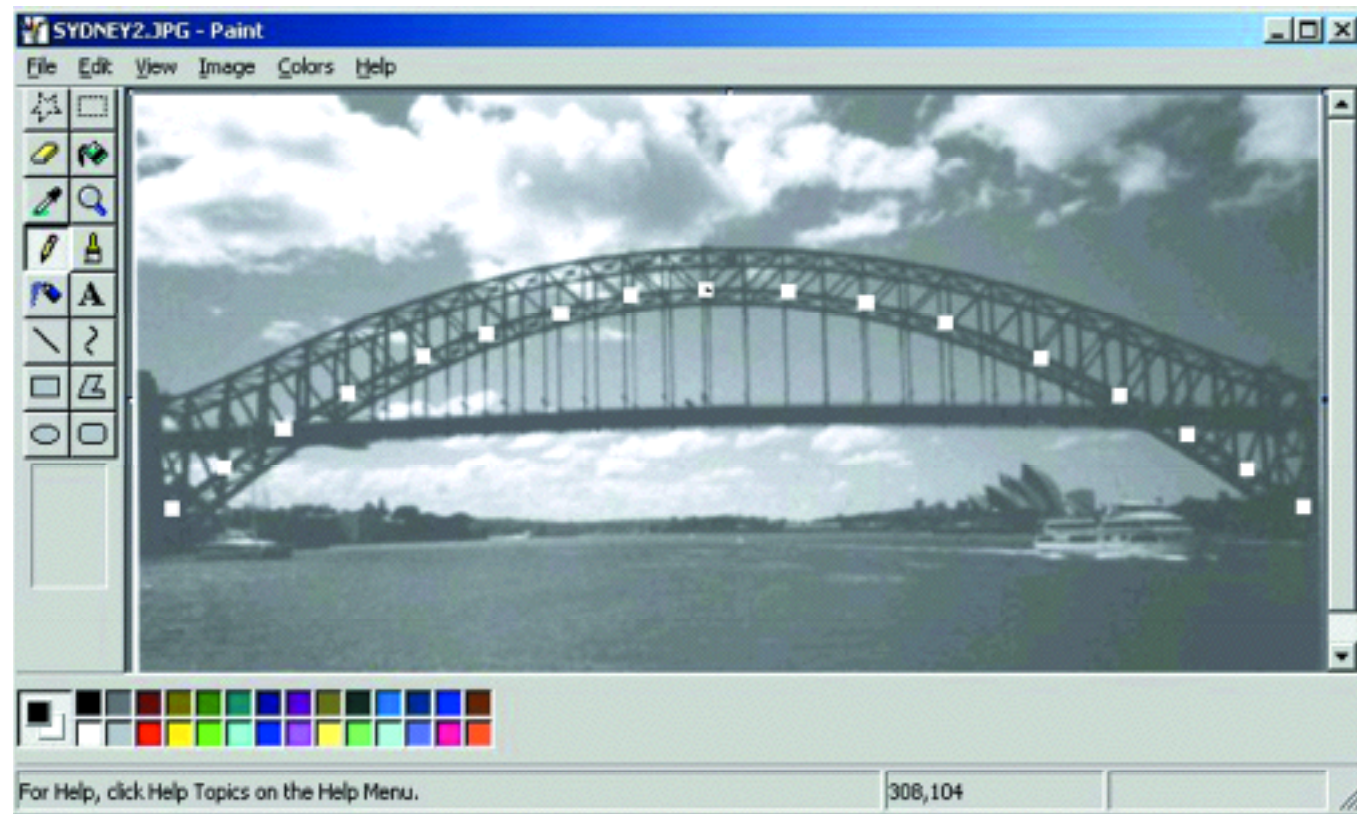
Adrian Oldknow

Figure 1

In an earlier article I looked at how image files, such as a JPEG file from a digital camera, can be imported in dynamic geometry software and how both geometric and algebraic tools can be used to superimpose objects, loci and graphs. In this article, we look at simple tools for digitising objects of interest from image files for treatment in other software such as graph plotters, data-handling software or graphical calculators. One such example is included in Oldknow and Taylor, (2000) and quoted in Appendix 13 of the Royal Society/JMC report on geometry (Royal Society, 2001). Here, a JPEG file has been imported into MS *Paint* (see Figure 1). The Eraser tool has been used to mark a few points on the lower arch. The cursor is currently at the central one of these, and the display shows that its pixel coordinates are (308, 104). Rather laboriously, we can either write down the full set of coordinates for the 17 points marked, or we could have, maybe, MS *Excel* also open and type them directly, one at a time, into this. So we have one, albeit rather laborious, means of digitising data from our image. Another snag is that the origin is in an awkward place – the top-left of the image, and the units, being in pixels, give very large numbers for coordinates. So some transformation of the data will be probably needed to carry out a sensible analysis.

Another approach could be to project the image directly onto a whiteboard (not necessarily interactive) or a large piece of paper on which a grid has been marked. Then you can mark points of interest with a board marker or felt tip pen, and read off co-ordinates from your own grid. This has the advantage that you can choose your own
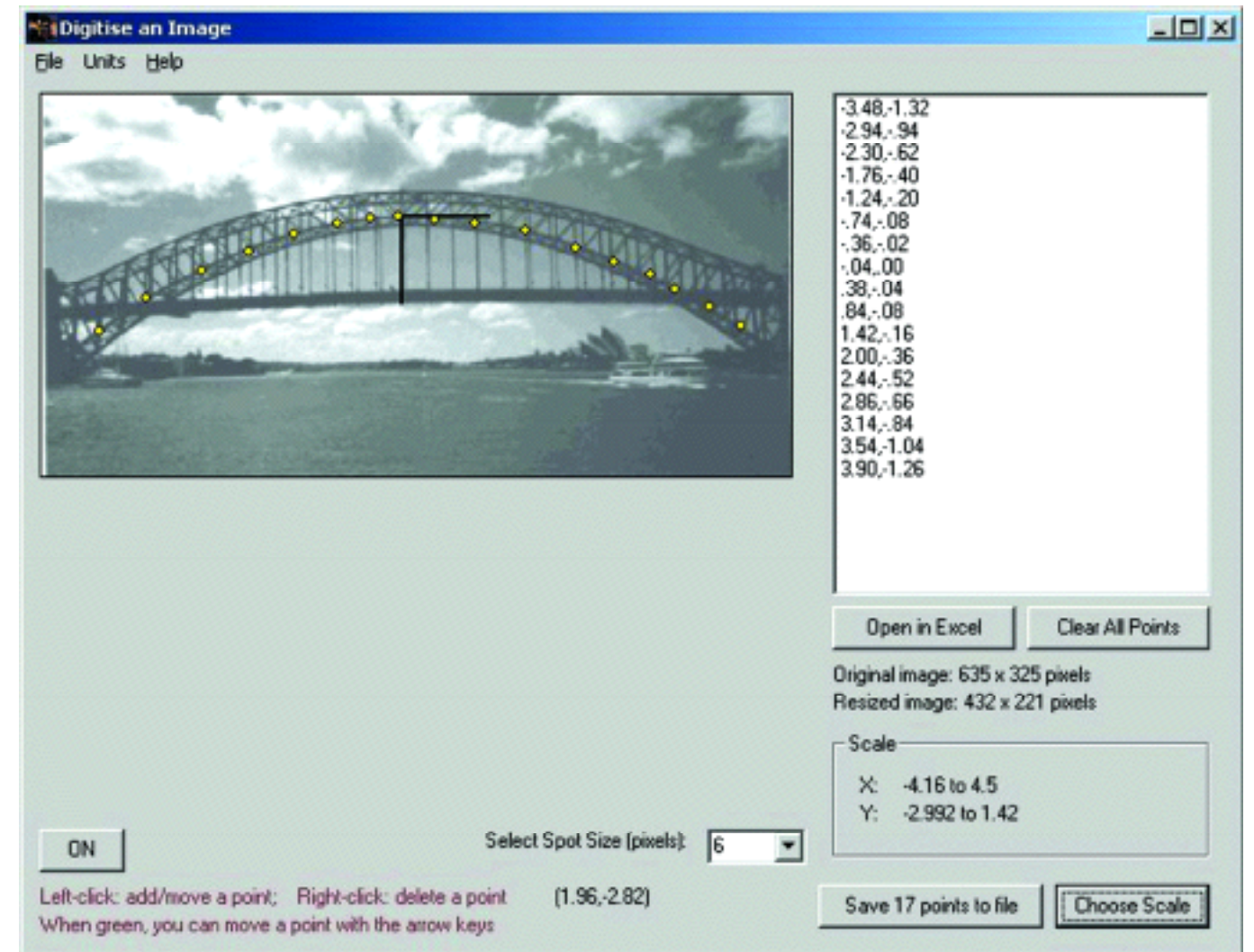

Figure 2

origin and scale. Fortunately, there is a simple ICT tool which we can use for the job, and which has the great advantage of being free! The *DigitiseImage* program (see Figure 2) allows you to define your own origin and axis scales. It was written by Jeff Waldock of Sheffield Hallam University and can be found at http://maths.sci.shu.ac.uk/digitiseimage/

Use the File menu to Open an existing image. Then use the `Choose Scale' button to define the origin and the scales for the x- and y-axes. Then, when you click on the image, a yellow point will
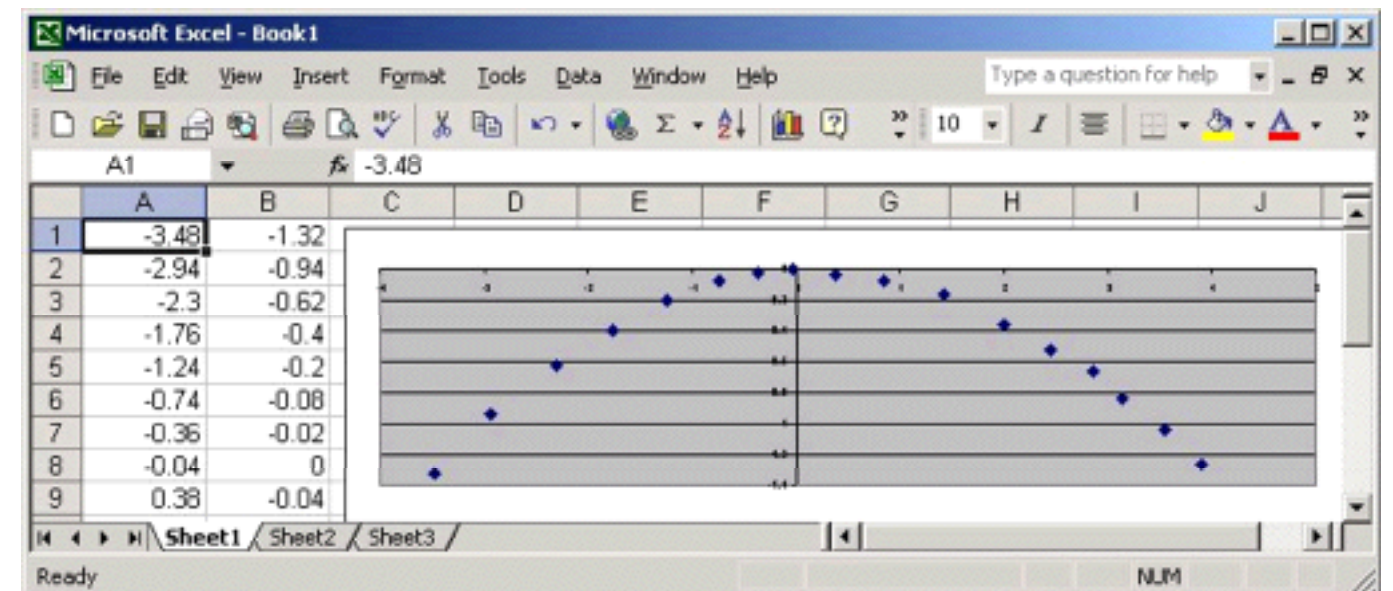

Figure 3

appear and its coordinates are entered into the data lists. You can move or delete any point. When you have collected your data, you can wither export them as a CSV (comma separated variable file), or directly into MS *Excel* (see figure 3).

One drawback with graphing from data in Excel is that we need the units on the axes to be of equal lengths, but we can only control the graph's dimensions by dragging the sides or corners of its box. A better tool would be to use something like *TI Interactive!* (TII) or, maybe, a graphical calculator. So we can copy the data from *Excel* and paste it into the Data Editor of TII (see figure 4).
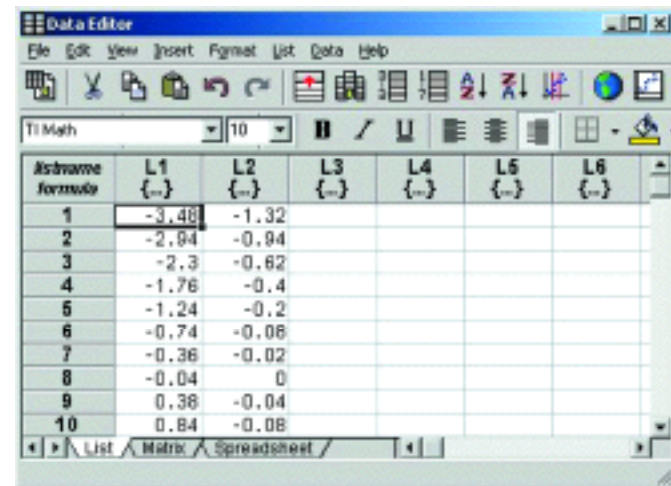

Figure 4

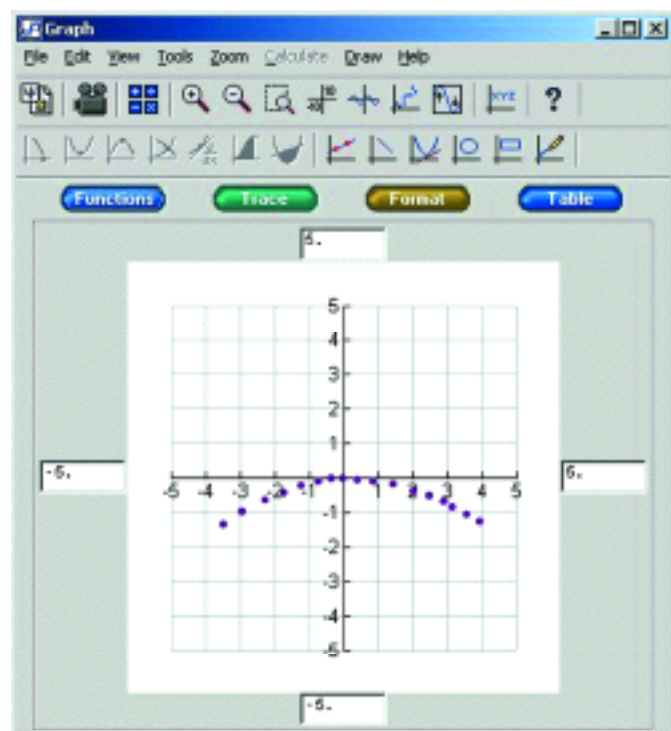Using the Graphing Tool, above, we can plot the scattergram from lists Lists L1 and L2 (see figure 5) and zoom in.

Figure 5

Again, from the Data Editor, we can fit a statistical model to the data – such as a quadratic regression (see figure 6):
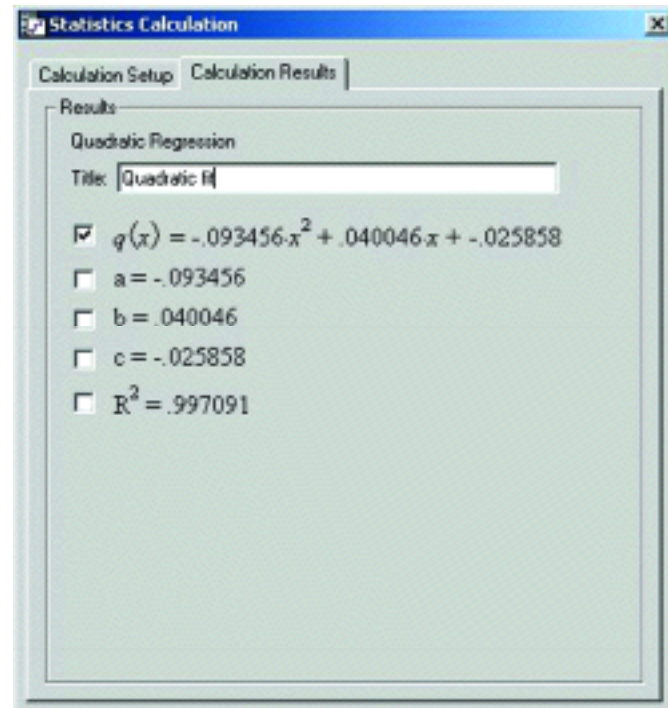

Figure 6

Here, we have a very high correlation coefficient, suggesting an excellent fit! (See figure 7.)
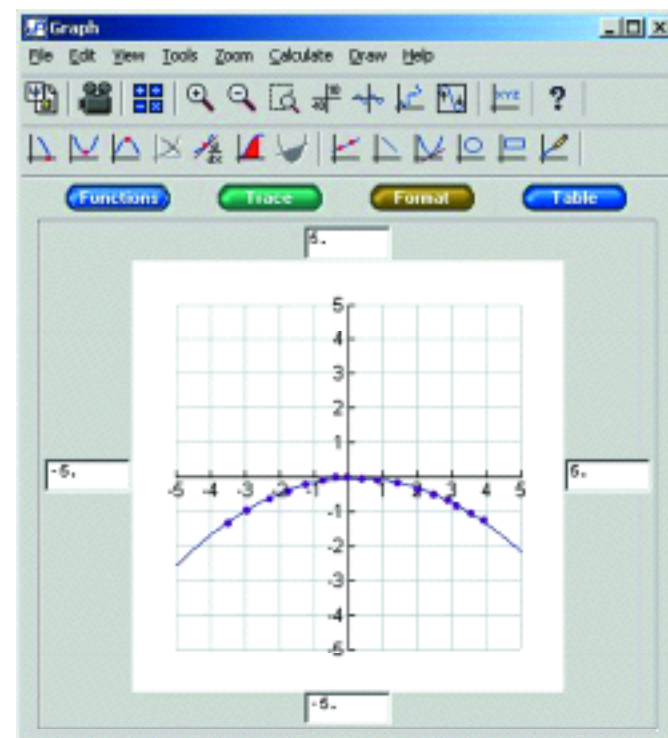

Figure 7

A very interesting development is now made possible with digital camcorders. This is the capturing of video clips from actions involving pupils themselves, such as in PE or sports lessons.

These can be transferred as so-called 'avi' files, using standard imaging software such as MS *Windows Media Player*. As before, the images can be projected onto a large display on which a grid has already been drawn. You can single step through the video clip and mark the position of an object. From its frame number and frame rate (say 25 frames per second) you can estimate the x-, y- and t- values for your object. Moving a few frames at a time you can build up, manually, a data table tracking the position of your chosen object against time. As with the still images, it would be very helpful to be able to utilise such data from movie clips using suitable software. For example, software called *Vidshell 2000* by Doyle V. Davis can be downloaded free from:

http://webphysics.tec.nh.us/vidshell/clips.html

This provides on-screen tools for the capture of time and co-ordinate data from video clips, as well as a library of useful video clips, such as the file *Basket.avi* used here (see figure 8).

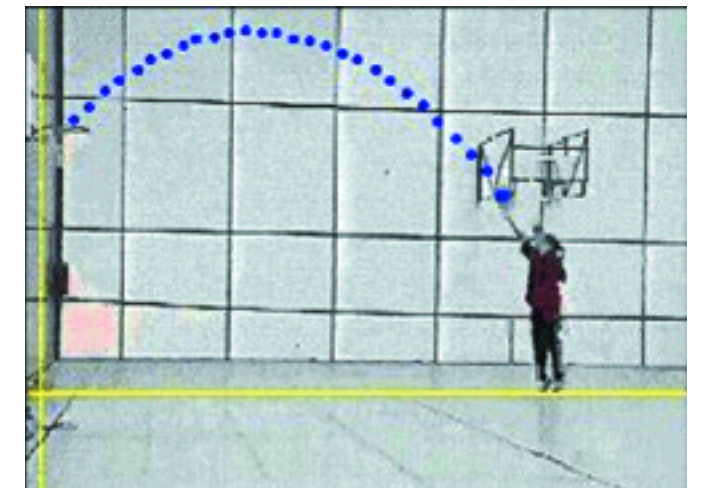First you open a clip with the Camera icon. This

Figure 8

appears in the movie window at the top left. Use the Rewind button to get back to the first frame. Now you calibrate the clip by using the software's Mouse icon, and drag open a segment of known length – such as the height of the basket above the ground: 3.05m. You can also use the Axes icon to define an origin and the positive direction for each axis. You will also be prompted for the time interval between frames – 0.04 secs in our case.
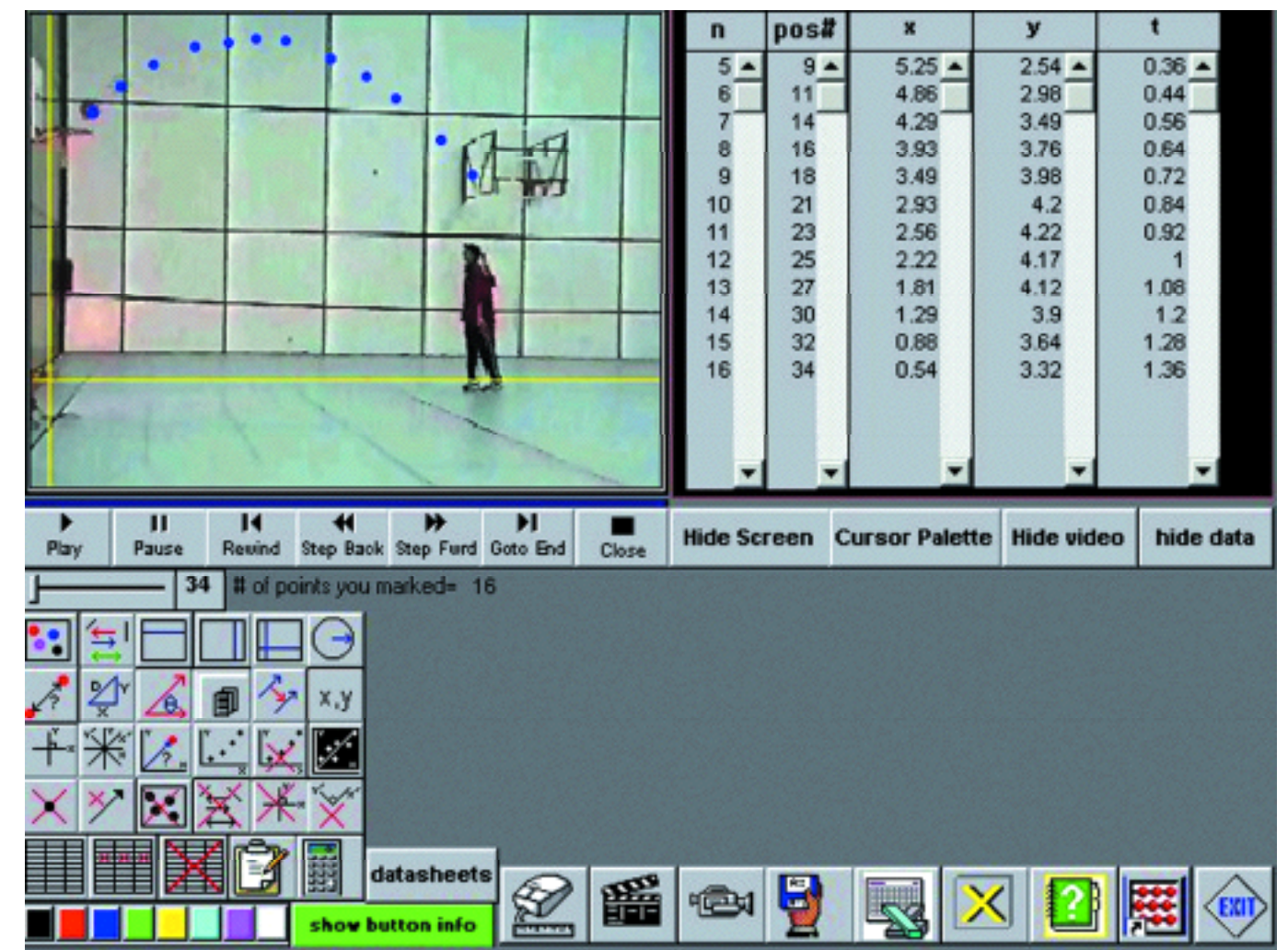

Figure 9

Opening up the Data window in the top right-hand corner you can now step the clip forward until the ball has been released and click the mouse where the cross-hairs coincide with the centre of the ball. This has two side-effects. It puts a coloured dot on a virtual `transparent overlay', and records the frame number, together with the x-, y- and t-values to the data table. When you have finished annotating and data-collecting you can export the data as a CSV text file, or directly into MS *Excel* (see figure 9).

Again the *Excel* route has the disadvantage of lack of control over the dimensions of the graph window.
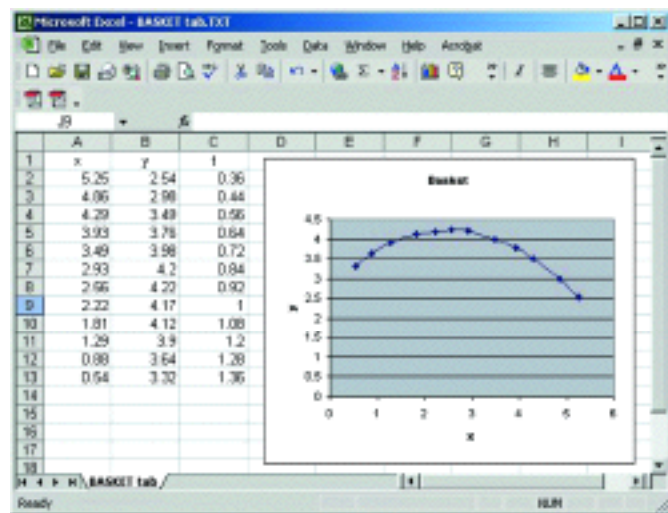

Figure 10

Using, for example. Word, you can do a Search and Replace to turn the commas into tab characters and save the data as a Tab delimited text file (see figure 10). This can be read into, for example, TI Interactive! Graphs and analyses can be performed on either *xy*-data, *xt*-data or *yt*-data. In particular, the *yt*-data should conform to a quadratic model where the coefficient of t^2 is $-g/2$, providing us with a means of estimating the acceleration due to gravity.

Of course the data can be easily exported to a graphical calculator such as the TI-83 Plus from which the following screens (see figure 11) were obtained for the *xy*-data.

Figure 11

The techniques and the software illustrated here are used in two lessons on quadratic functions which are part of a pack of resource materials supporting both ICT and mathematics at Key Stage 3 which have been developed by a joint ATM/MA/NAMA group for the Key Stage 3 Strategy, and which should be available from their web-site in the New Year.

Adrian Oldkno w t.

References
Oldknow, A. & Taylor, R.(2000) *Teaching Mathematics with ICT*. London:Continuum.
Royal Society (2001) Teaching and Learning geometry 11-19.London:Royal Society

Simple animated spreadsheets

John Sharp

One of the beauties of using spreadsheets is that it is possible to set up calculations,change some cell values and look at the effect on the results. What is not commonlydone is to make changes automatically and animate graphs. The example describedhere shows transverse and longitudinal waves, but the method has many other uses.The output is built up in steps to show how each part fits together, moving from apurely manual change to animation at the last step with a macro having only a fewlines. I believe teaching is about seeding ideas not showing how to do something once, so I see this as an idea that can be developed rather than an end in itself, which is why there are some challenges at the end. It is not a complicated technique, in fact it is one of those "easy when you know how" ideas that I hope will spawn further more novel animated graphs. I have assumed that you are reasonably proficient in

Sine waves

using Microsoft Excel, although this is not a highly sophisticated set of data.
Sine waves have many guises from equations to transverse or longitudinal vibrations. The exercise I am going to work through involves its appearance as each of these in turn to end up with an animated moving wave. So starting with the equation:

$$y = a \sin(x) \qquad \text{equation 1}$$

where x is the angle in radians, this would just give a static wave and in order to animate it later, it is necessary to add a phase offset (p). Changing this phase will create a travelling wave.

$$y = a \sin(x + p) \qquad \text{equation 2}$$

The graph of this function is a transverse wave as shown in figure 2. The moving wave when animated will be as if one end of a rope is being moved up and down which occurs when the phase changes.

A longitudinal wave is when movement occurs in the direction the wave is travelling. Such a wave occurs with sound alternately compressing and decompressing the air. The function which gives the x coordinate to be plotted is:

$$x + a \sin(x + p) \qquad \text{equation 3}$$

They coordinate is the same for every point.

Setting up the spreadsheets

The spreadsheet is set up as follows. I have used a technique of specifying angles by point number and then factoring them to get an angle. The technique will be obvious if you follow the steps, and has been done to simplify plotting angles. Please note that the cells have been chosen carefully and the large gap before the data at the left is to place the graphs.

1. In cell L2 enter the value 36. This is the number of points to be plotted.

2. Next to make cell M2 equal to • using the function PI().

3. In N2 create an angle factor from the expression =2*M2/L2

4. In M2 enter a value for the amplitude of 2

5. Now create a column which defines the point number, which is the numbers from 0