

Tilings and Tessellations

The activities are related to the work in Ma3 Shape, Space and Measures.

Pupils should be taught to identify and use the geometric properties of triangles, quadrilaterals and other polygons to solve problems explain and justify inferences and deductions using mathematical reasoning.



Organisation of the materials

The SMART Notebook™ software file is saved as "Tesselation.xbk".

It consists of 8 pages of which the first is the title page.

There are 5 pages to support the activity and its extension.

Page 7 is a blank page.

Page 8 contains teacher notes which are amplified here.

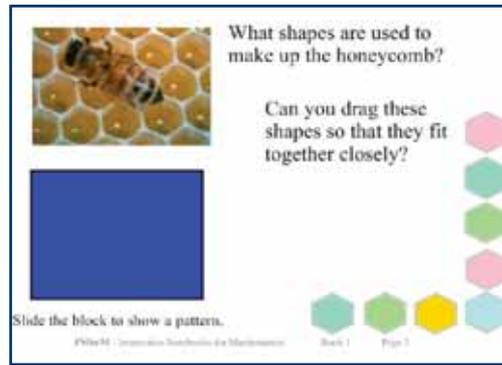
The Year 8 column on page 189 (Y789 examples) of the Framework suggests some practical activities.

Notes

The first activity

This is a very quick starter to encourage pupils to recall the name 'hexagon' or better still, 'regular hexagon' for the shape of the honeycomb cells.

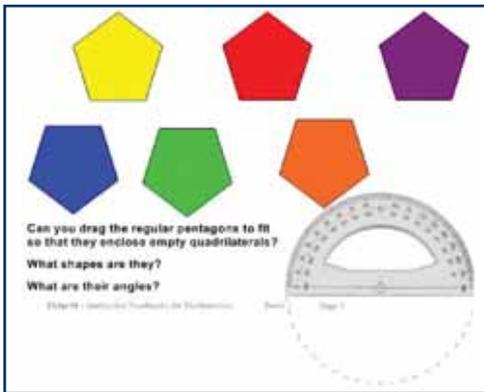
The stock of identical hexagons are provided for pupils to come up and drag together to fill an area of the board to show the way in which regular hexagons will tile the plane (i.e. tessellate). The blue rectangle hides a prepared collection of hexagons already 'glued' together. You might like to ask: "What is the angle (120°) at each vertex of a regular hexagon?" and hence justify why three hexagons can meet at a point without overlapping and without leaving any free space.



Page 2

The main task

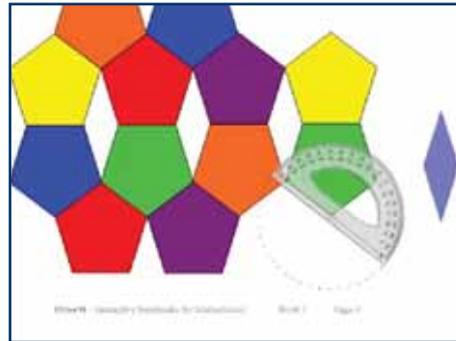
Page 3 of the SMART Notebook software file contains a set of regular pentagons. You might like to ask: "What is the angle at a vertex of a pentagon?" (108°), and hence explain why three pentagons will meet at a point but will leave some space.



Page 3

Ask pupils to try to visualise the shape being put together and get them to suggest some possible sorts of quadrilaterals they know e.g. square, parallelogram, rectangle.

Ask a volunteer to come up and drag the shape to fit as well as they can. The 'holes' should be rhombuses. Ask: "Can anyone explain why the four sides must be equal?"



Page 4

Page 4 shows a completed tiling. You can slide the protractor over the shape and also tap on it (or left-click) to reveal a green circle above it, which you can use to rotate the protractor to measure an angle - as you would the real thing.

The acute angles should be $360^\circ - 3 \times 108^\circ = 36^\circ$ and the obtuse angles $180^\circ - 36^\circ = 144^\circ$.

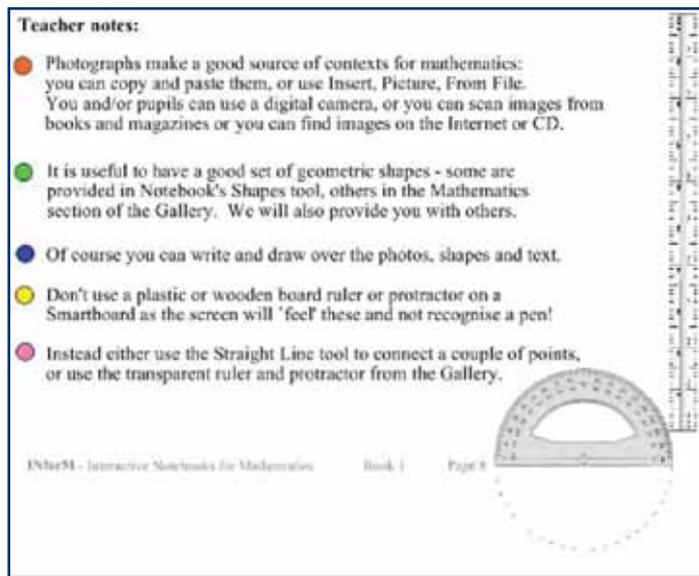
You can drag the pale blue rhombus over the gaps to show it makes a snug fit.

Thus we have established, using geometric reasoning, that a tiling pattern can be made from regular pentagons and a special type of rhombus – one with a 36° angle.

Notes

SMART specific

The SMART Notebook Gallery has an Education area, within which there is a Mathematics area. This contains some useful objects, such as the protractor and ruler, which you can drag into a SMART Notebook page, resize and manipulate.



Page 8

You can modify the "Tesselation.xbk" file in any way you like. If you do so, then save it with a different name in case you want to access the original again at some point.

The SMART Board™ interactive whiteboard is a touch-sensitive, analog device. If you try to use a conventional ruler or protractor in contact with the board it will produce unwanted 'squiggles' and you won't be able to draw with the pen. So either draw free hand lines, use the straight-line tool in SMART Notebook or use the 'virtual' ruler to guide your hand.

How to: create and use shapes in SMART Notebook

The simplest way is to use one of the prepared shapes on the SMART menu. For example, select the pentagon. When you move over the board a cross-hair will appear and when you tap it, will leave a small pentagon. You will stay in 'shape' mode until you tap the 'select' tool. Then you can drag the pentagon around. When you tap on your pentagon shape you will see some buttons to enable you to resize and orient the shape. The white circle at the bottom right allows



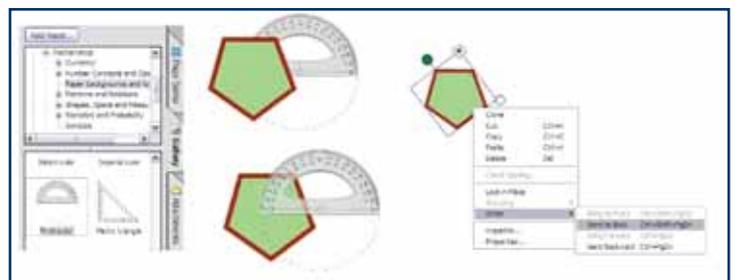
you to drag it to increase the size. This can change either or both the dimensions of the box surrounding your shape. So if you want to keep the figure as a regular pentagon use the SHIFT key while dragging and this will maintain the squareness of the box.

To rotate the shape, click in the green circle and drag it round. Click on the little box with the arrow to open up the menu. If you select 'Properties' then you can change the thickness, style and colour of the outline as well as selecting the colour to use to fill in the shape. If you want, you can also select a degree of transparency so that when you drag it over another shape, it will allow you to see the shape through it.

So here is an enlarged, rotated regular pentagon with a thick brown border and a pale green interior. It is not transparent.



In order to insert a protractor, select the 'Gallery' and find the Mathematics section of Education. Select the 'Paper backgrounds and tools' file and when this has loaded, scroll down to find the protractor. Drag it onto the current page.



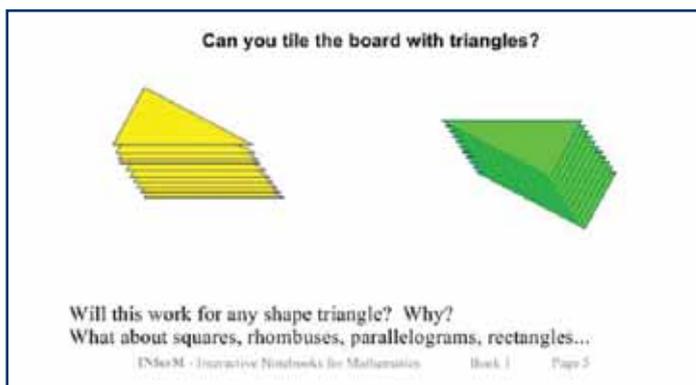
You can resize the protractor by clicking on it and using the 'shift-drag' technique to change its size without changing its shape. Sliding the protractor across the pentagon you will see that it 'hides' behind it. This is because the pentagon is currently in the foreground of the page. Right click on the pentagon and select 'Order' from the drop down menu. Now you can select 'Send to Back' and then you should be able to drag the protractor over the top of your pentagon.

Notes

The extension task

A filled scalene triangle has been created using some dynamic geometry software. This has been copied and pasted into page 5 of the SMART Notebook and 'cloned' to make a stack of identical yellow triangles. The original triangle has been rotated through 180° , filled with a different colour and then given the same treatment to produce the stack of green triangles – identical to the yellow ones but differently oriented!

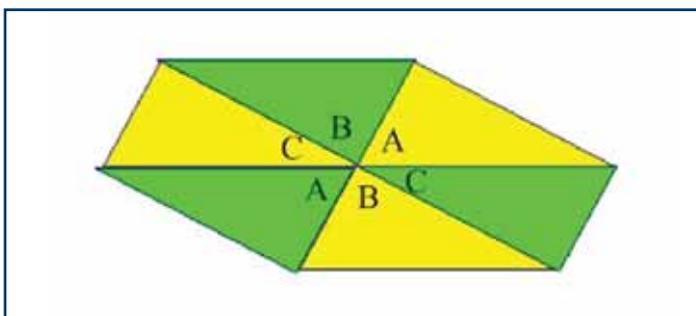
Before inviting any pupils to work on the board, ask the class to visualise the shape formed by putting a yellow and a green triangle together on opposite sides of a common edge (A parallelogram). Then ask how the yellow/green shapes might fit together.



Page 5

The key to explaining why any triangle must be able to tessellate the plane is shown below. If A,B,C are the angles of the triangle then we can always arrange six triangles around a vertex so that there are two of each angle opposite each other as shown. Since we know $A+B+C = 180^\circ$ (angle sum of a triangle), then the angles around any point must sum to $2 \times 180^\circ = 360^\circ$ i.e. six triangles completely encircle the common vertex.

Tessellating Triangles



Resources

Jones, K. (2005), 'Using Interactive Whiteboards in the Teaching and Learning of Mathematics: a research bibliography,' *Micromath*, V20(2) pp 5-7.

Nuffield Foundation (2005), 'Enhancing Mathematics Teaching Through New Technology. The Use of the Interactive Whiteboard.' *Mathematics in School*, V34(3) pp 4-6.

Ideas for follow up work

Page 6 shows two examples of photographs of tiling patterns. The one on the left was taken by one of the authors in Chichester Cathedral and shows a pattern of octagons and squares. The one on the right, of a building in Italy, was taken from some of the sample pictures which can be found on the 'Problem Pictures' website created by Richard Phillips:

<http://www.problempictures.co.uk/examples/op01.htm>



You will need to remind pupils that photographs such as these, taken with a camera, are perspective images so the shapes appear to get smaller as they leave the eye.

Page 6

There are many ways to capture digital images to use in software, such as SMART Notebook. You can use a digital camera (or one built-in to a mobile phone) and download them to your computer as 'jpeg' files. You can hunt for them on the Internet e.g. using Google Image search. The photograph of the bee with the honeycomb on the title page was found at:

<http://www.agr.state.il.us/programs/bees/photos.html>

- the website for the US State of Illinois Department of Agriculture – using a Google image search on "honeycomb".

Google Search



Another good source of information and images is the Wikipedia found at:

http://en.wikipedia.org/wiki/Main_Page

You can buy libraries of images on CD-ROM (such as in Microsoft® Encarta). You can also find an image in a book or magazine and scan it into your PC. Better still, you can encourage your class to provide images. A good source of tiling patterns is found in the Moorish architecture in Spanish cities such as Seville and Granada.

Digital images can also be imported as the background in dynamic geometry software, such as "Cabri II Plus" or the "Geometer's Sketchpad" (version 4) – and then you can perform constructions or draw graphs etc. over them.

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