What a picture, what a photograph

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The picture illustrated in Fig. 1 shows a Singapore landmark called Merlion. The statue is also a fountain sending a water spout into the Singapore river. What information can we glean from this simple snapshot?

A starting point might be to guess that the path of the water spout is a parabola; in theory it is made up from lots of little water particles which emerged from the spout at different time intervals. Providing there is no turbulence, or strong wind etc. then each particle should behave like an ideal projectile whose position can be determined from the initial velocity, which we'll assume to be horizontal at the moment of leaving the Merlion's mouth.

In order to carry out some modelling we are going to need to take some measurements from the image, and to do this I am going to use dynamic geometry software to show the modelling power of tools such as the latest versions of the *Geometer's Sketchpad* (Version 4) and *Cabri Geometry* (II Plus). The *Sketchpad* version is shown in Fig. 2.

The segment AC has a point B on it so that B can be slid to define a unit distance AB, a technique known in the trade as a 'slider'. The point O is taken at the opening of the Merlion's mouth. O and AB are used to define the coordinate system. The parameter a_1 is defined with an initial value of -0.25, but can be easily varied. The function $\mathrm{f}(x)$ is defined algebraically in terms of both the parameter a_1 and the ordinate x. *Sketchpad* has a function which will directly plot the graph of a function and this produces the parabola as shown in Fig. 2. Varying the point A we can make this as close a fit as we can 'by eye'.

Also in the image in Fig. 2, we have defined a point Px on the x-axis, and constructed the corresponding points Py on the y-axis and P on the curve (dragging Px to make P as good a match as possible to where the water spout hits the river). In order to be able to relate the image to real life we need to know at least one length of an object in the scene. Fortunately, a search on the Internet soon reveals a report from the 'Straits Times' of the re-siting of the statue, whose height is given as 26 feet. Assuming, in this case, that we can trust what we read, then we can take a measurement MN from the image and compare it with our unit distance AB in order to work out a conversion factor between centimetres in the photograph and metres in Singapore. From the data in the image we find that one *Sketchpad* unit represents 27.6 feet or 8.41 m. Finally we have constructed a point Q on the y-axis and the ray QP, so that we can slide Q to get as close a possible fit for QP to the tangent at the point of entry of the water to the river—an angle of about 47° with the horizontal.

Fig 1. Merlion.

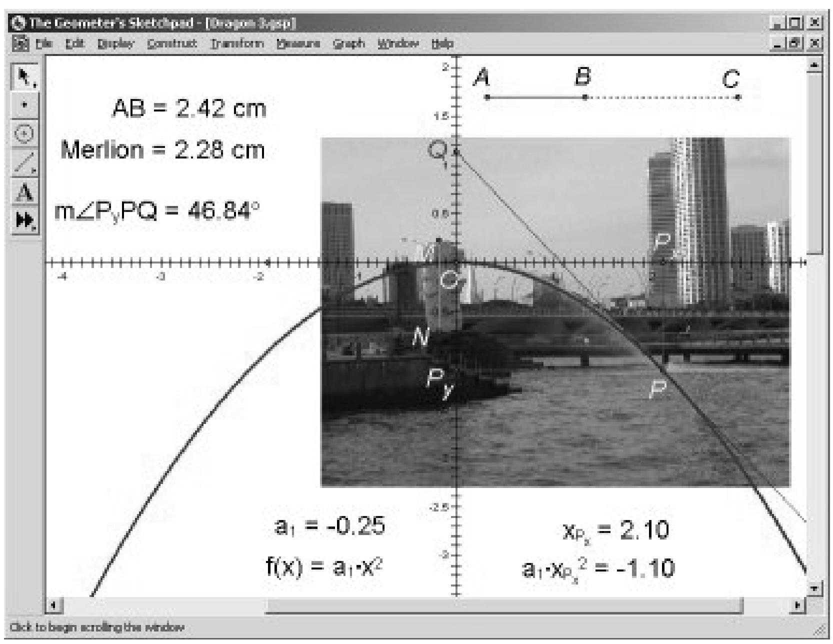

Fig 2. Modelling the path of the water spout from Merlion using *Sketchpad.*

In Fig. 3 we show the similarly produced model using *Cabri Geometry II Plus*. Here we have to construct the point *Px* first in order to substitute its x-coordinate into the formula for f(x). The result is used to construct *Py*, and hence *P*. The locus of *P* with *Px* then gives the required quadratic graph. Again we can slide *B* on *AC* to make the given quadratic as close a fit to trajectory as possible.

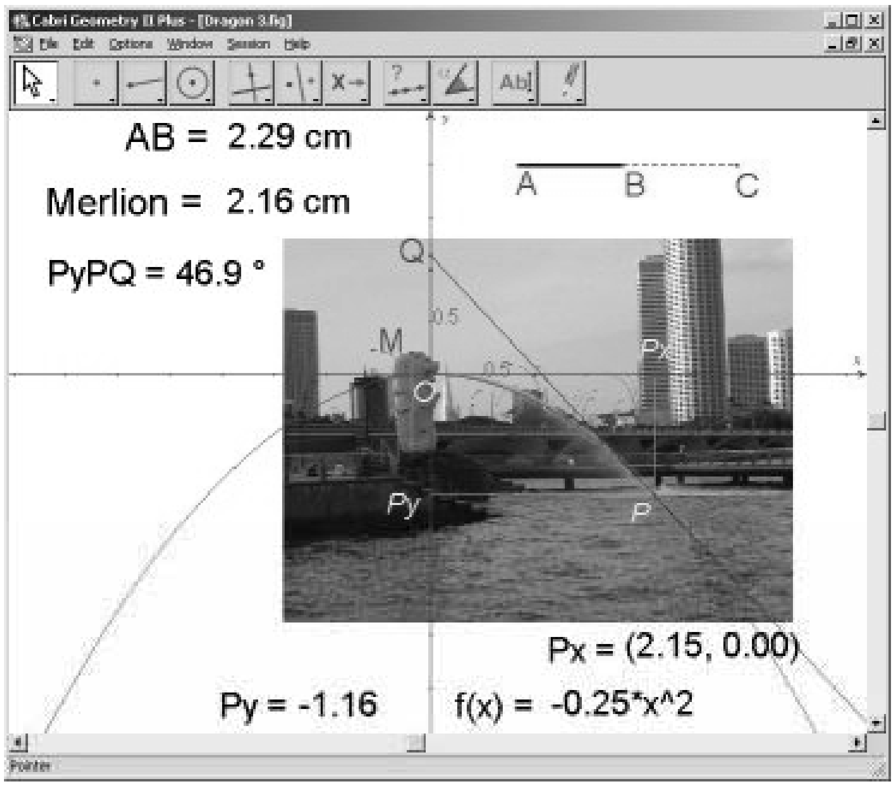

Fig 3. Modelling the path of the water spout from Merlion using *Cabri Geometry II Plus*.

Although the actual measurements of MN and AB differ slightly between the *Sketchpad* and *Cabri* models we find that their ratios do correspond and that the *Cabri* unit also corresponds to 8.41 m.

Now we just use some elementary mechanics to relate the theoretical trajectory of a water particle to our model of $\mathrm{Y} = -\frac{1}{4}\mathrm{X}^2$. If (x,y) denotes the position of P in metres relative to O at time t seconds then we have: $x = vt$ and $y = -\frac{1}{2}gt^2$ where g is the acceleration (in ms^{-2}) due to gravity and v is the initial horizontal velocity from the Merlion's mouth (in ms^{-1}). Eliminating t, we have: $y = g/(2v^2)x^2$ where $x = k\mathrm{X}$ and $y = k\mathrm{Y}$ with k the 'conversion factor' of 8.41 m per unit.

Substituting these we find: $\mathrm{Y} = -(gk)/[(2v^2)\mathrm{X}^2]$, which gives us the correspondence: $v^2 = 2gk$. Hence, using $g \gg 9.81\ \mathrm{ms}^{-2}$ we find that $v \gg 12.9\ \mathrm{ms}^{-1} \gg 46.5$ kph $\gg 28.9$ mph.

Given the uncertainties in our measurements and in the newspaper account, it would be remarkable if these were accurate to 2 s.f., but at least we have a 'ball park' figure.

From our geometry models it appears that the range of the spout is about 2.1 units or 18 m, so that the water takes about 1.4 seconds to hit the river. The gradient of the tangent at this point will be given by $\mathrm{dY}/\mathrm{dX} = (\mathrm{dY}/\mathrm{d}t)/(\mathrm{dX}/\mathrm{d}t)$.

But since $y = k\mathrm{Y}$ then $\mathrm{dY}/\mathrm{d}t = 1/k\ \mathrm{d}y/\mathrm{d}t$ and similarly $\mathrm{dX}/\mathrm{d}t = 1/k\ \mathrm{d}x/\mathrm{d}t$.

Also $\mathrm{d}x/\mathrm{d}t = v\,\mathrm{ms}^{-1}$ and $\mathrm{d}y/\mathrm{d}t = -gt\,\mathrm{ms}^{-1}$ so that $\mathrm{dY}/\mathrm{dX} = -gt/v \gg 1.06 \gg \tan 46.8^{\circ}$.

Again both our geometry models seem to square with this result.

Can you calculate the velocity with which the water spout hits the river at P?